

## STUDY OF LOGICAL RELATIONS USING TREES AND GRAPHS IN TASK WITH CLASSIFICATION TABLES

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### ABSTRACT

In this work, the symbolic diagrams are drawn up of the logical relations established by children of between 3 and 5 years old during the resolution of a classification task on a table using symbolic codes that was set out in two forms: as the application and as the discovery of rules. Empirical data collected from 211 children provide graphic explanations of the significative difference found in the results between the answers in direct and inverse mode task. In mathematics the coexistence of two different but interrelated processes of thought to be highlighted: direct and inverse. However, the tasks in inverse mode should be standard practice in the preschool classroom. Moreover, the great success at the task in direct mode, recommended in some textbooks for children between 7 and 9 years old, underlines that the programming of activities designed to build up mathematical thought process at this stage should be reviewed.

**Keywords:** Logical reasoning, graphs, relational mathematics, preschool education.

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## JUSTIFICATION

The constructivist paradigm, and, more specifically the Piagetian vision, identifies the construction of mathematical knowledge as lying in the practice of basic procedures such as classification, seriation, ordination or transformation (Flavell 1982). Thus, from that perspective, knowledge is not transmitted but it is constructed by the individual as a consequence of interaction with the surroundings and by acting on the objects.

Mathematics can be conceived of as a science which seeks to establish relations between facts and elements of very diverse types (Alsina, Burgués, Fortuny, Giménez, Torra, 1996). The analysis of two paradigmatic examples, in particular the progressive-regressive demonstrative method (Solow 1992) and procedures for analysis and synthesis introduced by the greek mathematician Pappus referred to by Polya (1984), enable the coexistence of two different but interrelated processes of thought to be highlighted in a diversity of situations in the field of mathematics, which we have termed direct and inverse (Ruesga 2004).

Direct and inverse processes are two forms of reasoning that the individual do. These can be characterized by the focus of each one. In the direct modes processes of the think, the focus is the beginning of the situation or the dates. In the other hand, the focus of the inverse mode processes is the results or the final situations. The two modes are continually mixed up in any resolute, constructive or demonstrative process and constitute stages of reasoning that together make up the complete process and are sustained by methods of logical deduction.

These mechanisms require that any operation, direct and inverse, to be integrated into one single operation, formal or otherwise (Piaget 1979). In this way, before the formal operation, the children can constructed semi-empirical abstraction, that is, the subject need to integrate the two processes direct and inverse about non-formal operations by observation the objects and their implicit relations.

This non-formal mathematics operation that characterizes the pre-school educational stage allows us, nonetheless, to attend to the demands that mathematics will place on them in the future with regard to direct and inverse relational processes.

In fact, the practice of those latter procedures is identifiable from the point of view of the two modes as tasks involving the application of express rules, in the case of the direct mode, and the discovery of rules, in the case of the inverse mode. In particular, a multiplicative classification task in direct mode consists of explaining the rules for classification by the rows and columns on a grid, through the use of symbolic codes that refer to the attributes or characteristics of the elements to which they are to be applied. In the case of the inverse mode, which leads from the effects to the causes, the task consists of discovering how the classification was made in the light of only some of the elements. However, the logical relational processes that take place in one mode or another, own of a one single task, could be different.

Even in the studies of Dienes (1987) as well as some texts for preschoolers propose tasks only in direct mode. We have found no work related to classificatory tasks in inverse mode with children of this age. We therefore designed some tests through which to analyze the behaviour of children, their chances of successfully completing the tasks, and the arguments they present confronted with each of the two modes.

The presentation of tasks relating to these procedures, in both modes, may be considered as an external stimulus that leads the child to put into practice relational processes of a logical-type through semi-empirical abstraction. These abstracts relations could be observed only when the children doing by concrete objects.

We propose in this paper to identify the logical relations that the subject establishes to solve the classificatory tasks in both modes: direct and inverse. We investigate through observation and analysis of the basic steps used in the corresponding resolutive processes and to represent these in symbolic form with a view to establishing the relational diagram that develops in the course of both processes. A comparison of the diagrams corresponding to each of the two modes will contribute to establishing a relationship between them that will allow us to explain our findings.

## **DESCRIPTION OF THE EXPERIMENT**

Our descriptive experimental study, of an exploratory type, was carried out with only one single measurement, with which a multivariate process analysis was performed. A total of 211 children participated in the research from typical preschool classes belonging to seven public and private schools in the city of Burgos, of whom 70 were 3 years old, 76 were 4 years old and 65 were five years old. All of them were individually interviewed and the session was recorded on video.

### The task

A multiplicative classification task inspired by the work of Dienes (1987) was designed, to be solved in two modes: by the application of express rules and by their discovery.

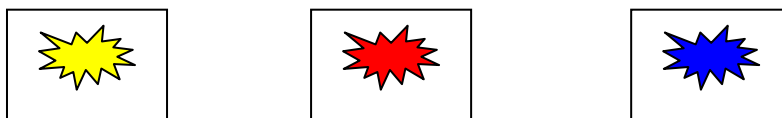
Specifically, in direct mode, it consists of placing six thin triangles from a set of logical blocks in their corresponding squares on a grid of squares whose rows and columns display cards bearing symbolic signs, respectively relating to the attributes of size and colour. In inverse mode, two triangles are presented on their corresponding squares. The task therefore consists of positioning the cards and the remaining triangles.

The cards are the symbolic codes indicative of the variables:



**Figure 1-** Symbolic codes indicative of the size

to indicate large and small sizes; and the cards:

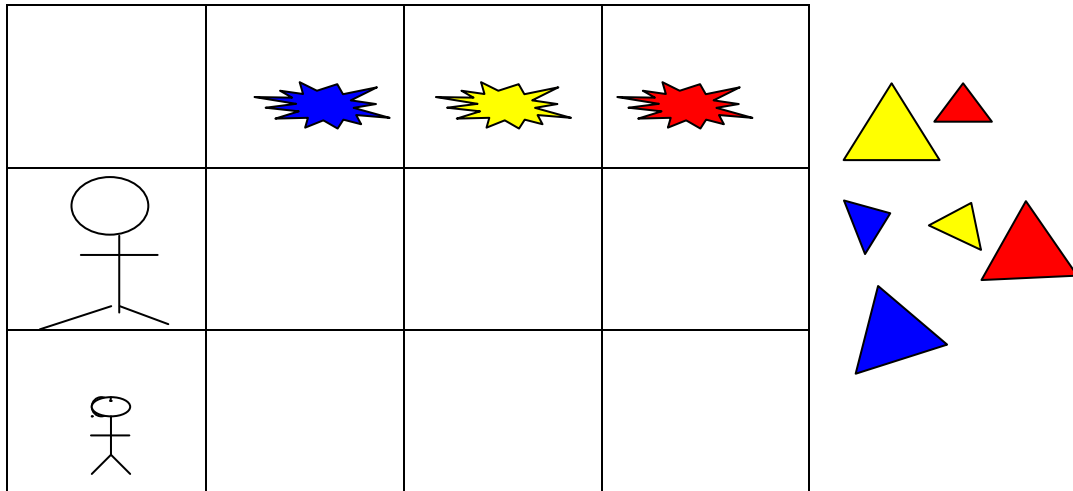


**Figure 2 -** Symbolic codes indicative of the colour

to indicate the three colours.

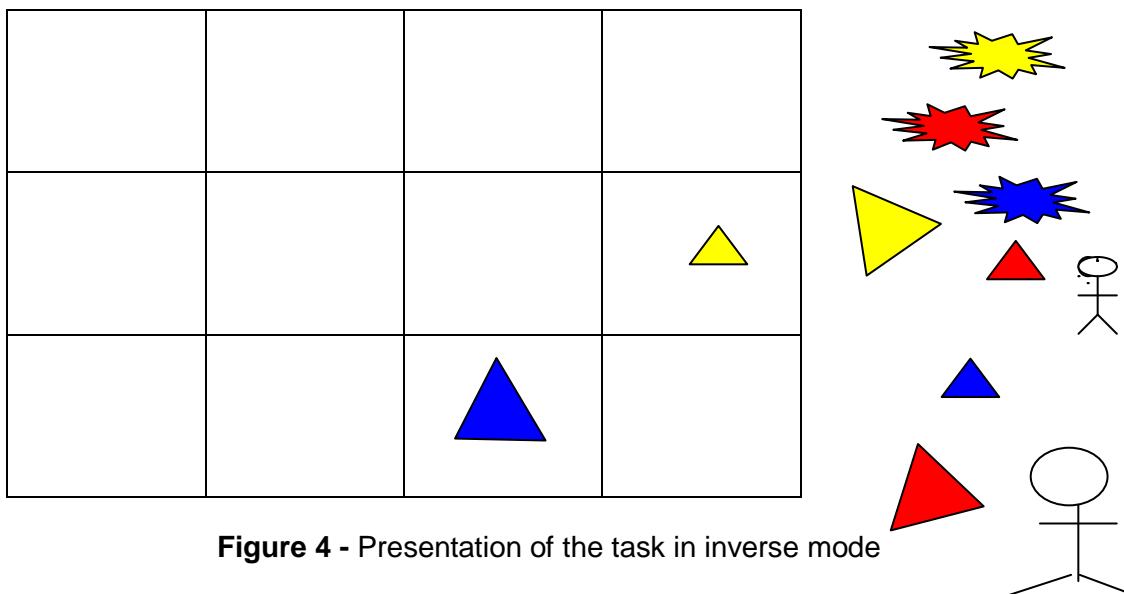
The task begins with an explanation of the meaning of these symbols, checking that the children recognize these variables on the thin triangles of the

logical blocks. Finally, the objective is explained and the grid is presented in the following form:



**Figure 3** - Presentation of the task in direct mode

The task in inverse mode consists of putting all the elements in place that are outside the grid and is presented in the following form:



**Figure 4** - Presentation of the task in inverse mode

### Procedure used for the test

The test began with the task in direct mode and after some others activities that we don't explain in this work, or 20 minutes later, was followed by the task in inverse mode. Video recording were made of the interviews.

The dialogues followed the script found in Annex I.

## RESULTS

In the following table, the *success rates* for successful completion of the tasks in both modes are shown by age groups:

<b>Success rate</b>						
(N = 211)	Direct mode			Inverse mode		
Age	N	% of group	% of 211	N	% of group	% of 211
3 years old	67	95.7	31.8	39	55.7	18.5
4 years old	74	97.4	35.1	65	85.5	30.8
5 years old	62	95.4	29.4	54	83.1	25.6
Total	203	96.2	96.2	158	74.9	74.9
Significance (95%)	$\chi^2 = 4.588$ $P \leq 0.598$			$\chi^2 = 27.189$ $P \leq 0.001$		

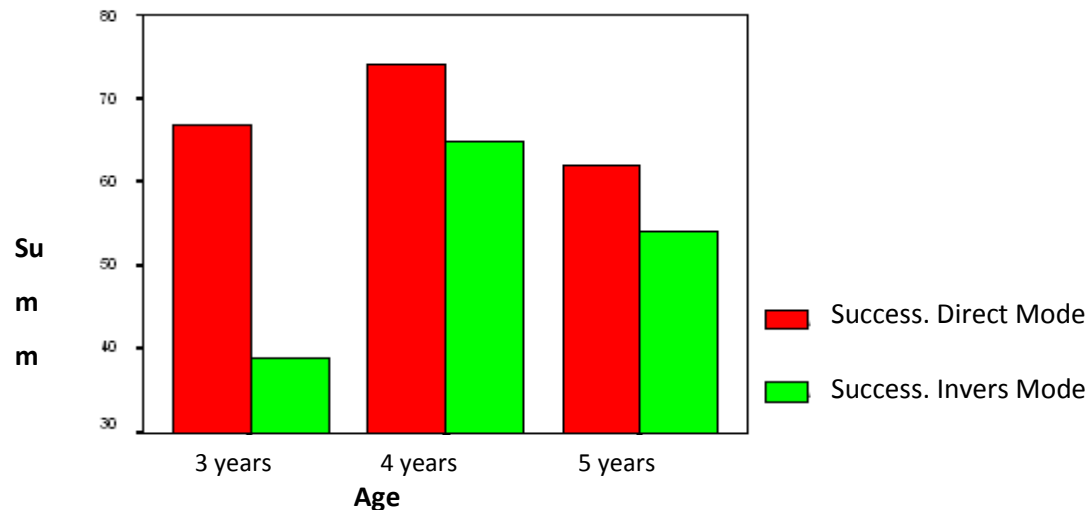
**Table 1** - Achievements in the classificatory tasks in the two modes and comparison of mean results by age

The task in direct mode is successfully solved by practically all the children of all ages though a significant difference in the success rates is found by age. Children of 3 years old attain percentage success rates in their actions that are equal to those of 5 year olds. This confirms that classification is one of the earliest logical-type activities to appear (Piaget 1979).

However, the task in inverse mode achieves significantly different success rates according to age. Although it is the group of three-year-olds that achieves the lowest success rate, that is over 55%.

Comparison of the success rates by mode allows us to affirm that the inverse mode is significantly more difficult to solve than the direct mode ( $\chi^2 = 17,204$   $P \leq ,001$ )

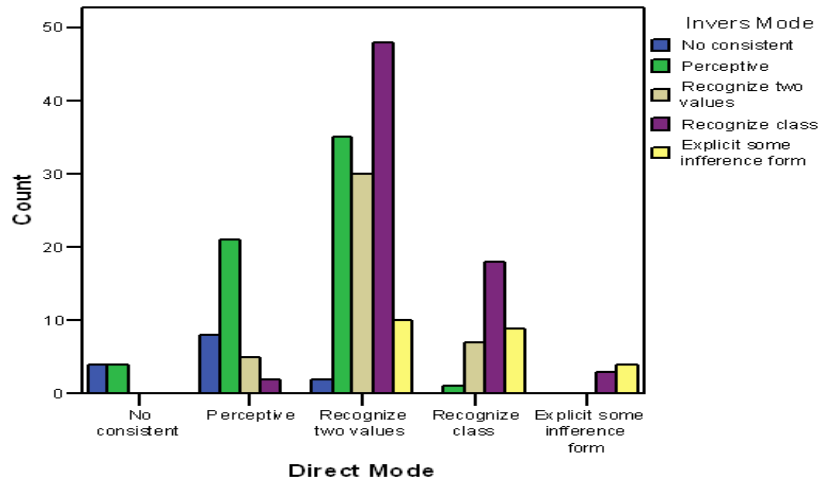
In the following table, the success rates for completion of the tasks in both modes are shown by age groups:



**Graph 1:** Classificatory task success rates in both modes by age groups

Among the different categories established to analyze the *arguments* that the children use in their actions, the most highly valued is that which is closest to inference, expressed using the future voice, for example *'because the reds will go here'*.

Comparing the argumentations in both modes we found a significant difference in the types of arguments employed ( $\chi^2 = 112.378$ ;  $P \leq 0.001$ ). The argumentative expressions considered as 'better' are more frequent in inverse mode. In addition, the same subjects will not always express their arguments to the best of their ability. Even though the child can put forward better arguments, if they are not used in direct mode then it implies that the task is less demanding and does not require as much effort. The following graph compares the argumentation in both modes.



**Graph 2 - Comparison of the argumental categories by mode**

The data shows a significant difference, in both modes, between age groups in the types of arguments employed by the 3-year-old children as opposed to those of other age groups (In direct mode between 3 and 4-years-old:  $\chi^2 = 24.495$ ;  $P \leq 0.001$  and between 3 and 5 years old  $\chi^2 = 30.536$ ;  $P \leq 0.001$ . In inverse mode between 3 and 4 years old  $\chi^2 = 47.345$ ;  $P \leq 0.001$  and between 3 and 5 years old  $\chi^2 = 52.602$ ;  $P \leq 0.001$ ). This group uses less elaborate categories of arguments with greater frequency than the other age groups and is distinguished, above all, by their use of argumentation based on perceptual types of reasons with expressions such as *'because this one's here'* (pointing with a finger to the pieces or elements already in place).

However, as we know, the difference in argumentative expression of the group of 3 year olds, does not suppose less success in the resolution of this group's task in the case of direct mode.

## RELATIONAL DIAGRAMS

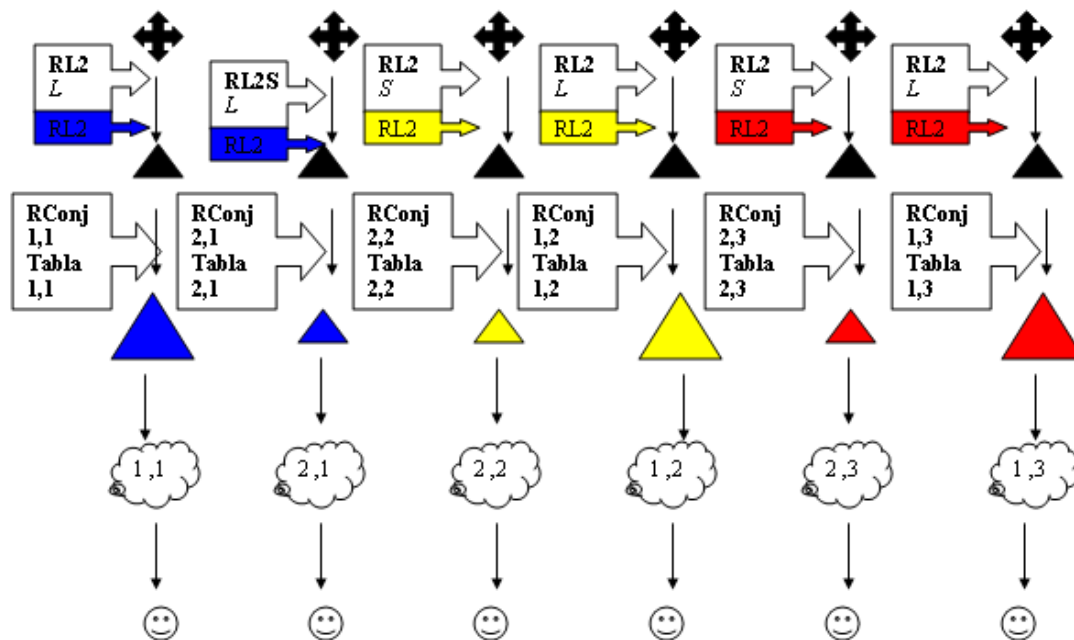
The different *procedures* used by the children to resolve the tasks show a significant difference, according to age, in the case of the direct mode ( $\chi^2 = 13,487$   $P \leq .001$ ) but this is not so in inverse mode. The relations that need to be established to resolve the tasks can be symbolized in the form of relational diagrams (For the



meaning of the symbols see Annex II). The fact that, in inverse mode, it is necessary to position more elements than in direct mode. In addition, those elements have different degrees of concretize (ones are concrete elements, like triangles, and others are more general- symbols of class, like targets of colour or size). Between the different resolutive methods possible to be identified, we can find someone that shows the inferential reasoning of the child. When the child begins with the red coloured card, they have successful, because this is the most general and occult element when the task is presented.

We describe below how some child answer the tasks and the logical reasoning they used during the resolutive process.

### How Daniel (5:2) solves the classification: Direct mode



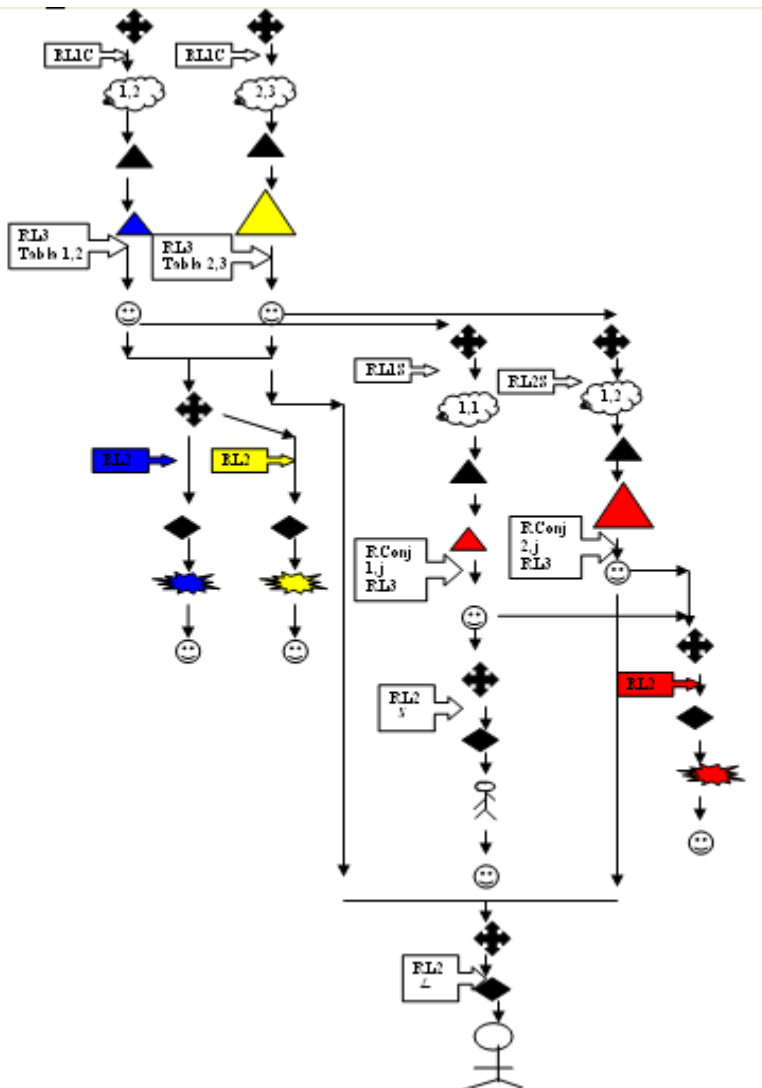
**Graph 3** - How Daniel (5:2) solves the classification: Direct mode

The child observes the grid, the triangles and establishes the relations (of the large blue triangle) belonging to the classes whose colour and size codes are represented by their respective symbols. He associates the connection of both attributes, identifies them in relation to the mentioned piece and identifies this element with the position (1,1) (first row and first column) on the grid. He then goes on to position the large blue triangle successfully in the correct place. The process repeats itself with the remaining pieces: small blue triangle, small yellow one, large

yellow one and large red one, there by completing the grid, by columns, according to colour.

It is only this procedure that can lead to the right answer. The only possible variation to the procedure is the order in which the pieces can be transformed, for example, by starting with the smaller ones or another one, however the directionality used in our culture supports the order that is shown in this example. There are no other alternative procedures in any other sense. In this way, the latter tree is representative of the procedure that, with the exception of the previously mentioned variations, all the children use.

The procedure that is used to solve the transformation in inverse mode implies establishing the following relational networks

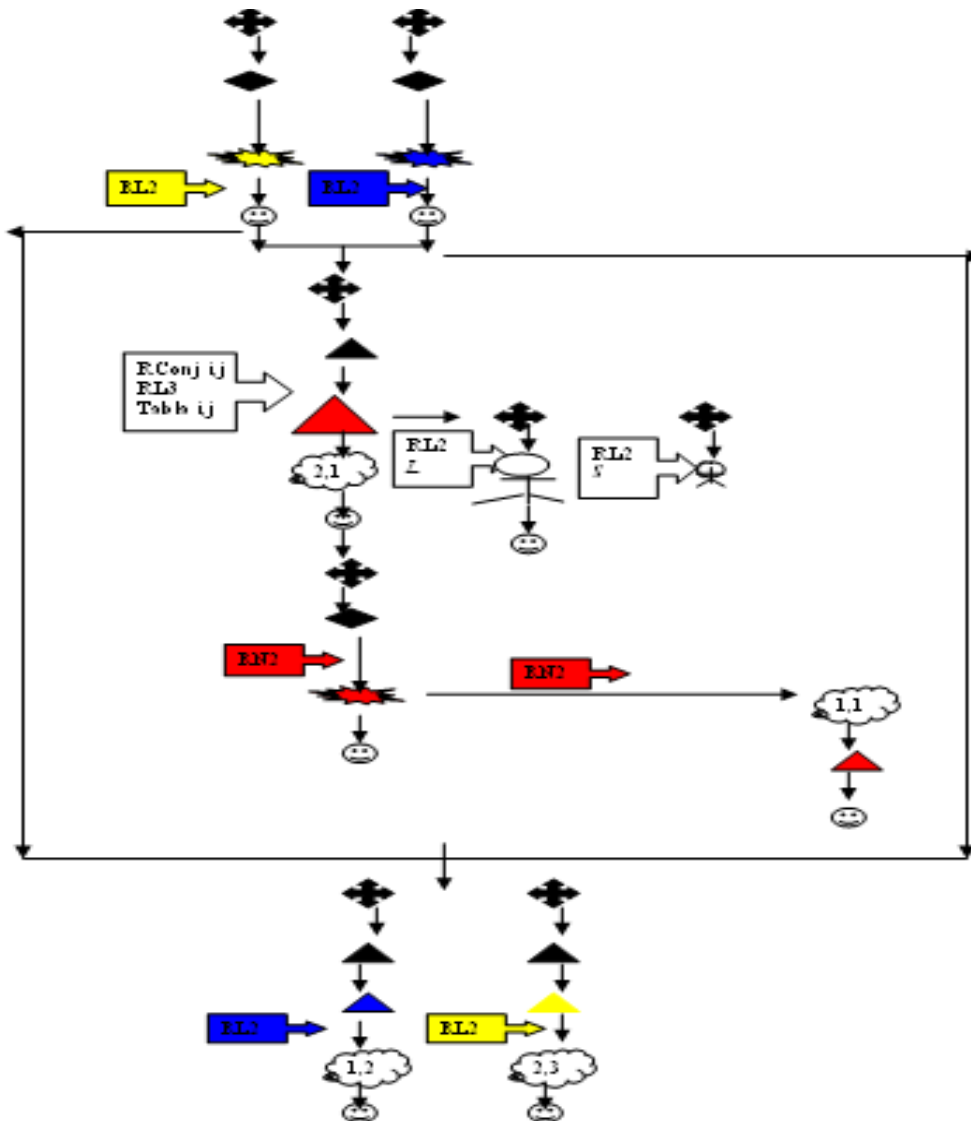


**Graph 4** - How Daniel (5:2) solves the classification: Inverse mode

After observing the situation, the child begins to establish a relationship of size and associates it with the relevant square of the grid before correctly positioning the large blue triangle, and continuing with the large yellow triangle and then with the colour cards that identify both columns. The child continues with the same process, positioning the two red triangles, after the red colour card and finally, the two cards indicative of size.

This procedure always begins with the most specific concrete elements (the triangles), whose place can be identified through basic strategies of perception of similarities. The most symbolic element that is the red colour card is only identified a little further on and with less initial data.

**How Inés (4:8) solves the classification: Inverse mode**

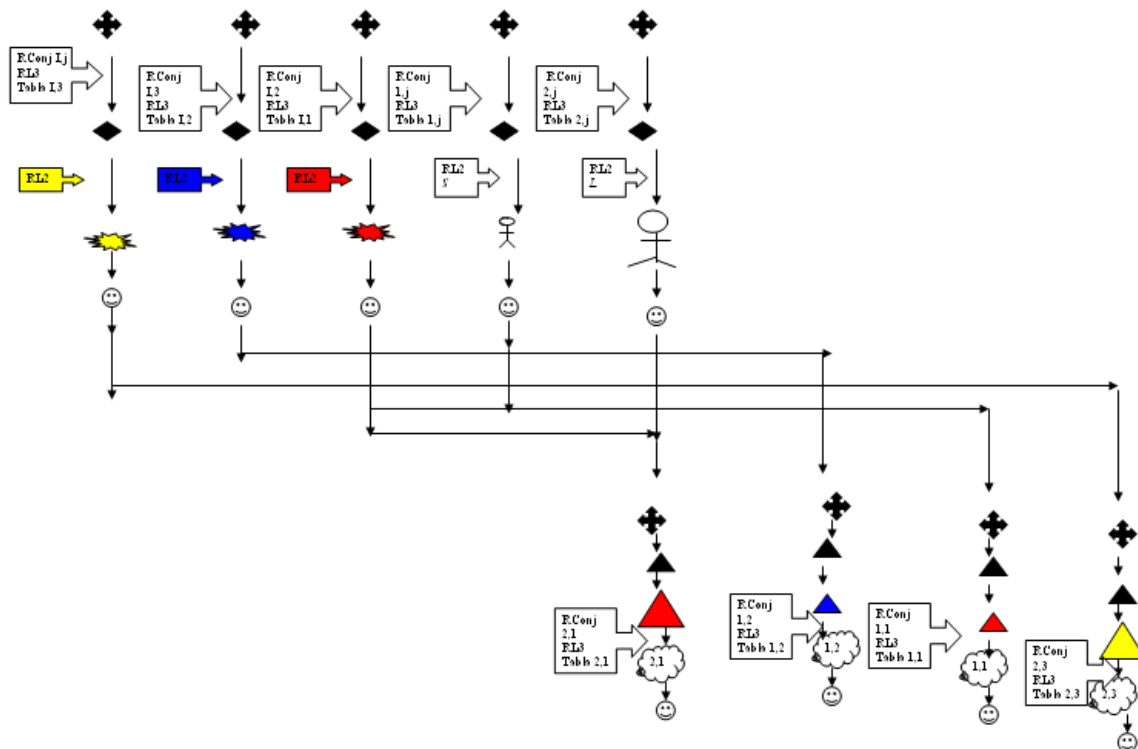


**Graph 5 - How Inés (4:8) solves the classification: Inverse mode**

The girl starts to identify and position in their respective places the cards that symbolize the colour (first the yellow one and then the blue one) corresponding to the pieces on which she has information. She observes the elements and identifies the place that corresponds to the large red triangle, and continues with the red card, the small red triangle, the card for large sizes, for small sizes and, finally, the small blue and large yellow triangles.

It could be said that the girl displays a procedure that is more detached from a perceptual one, as she starts by using the cards with the colour symbols, and it seems that the justification was the positioning of the large red triangle immediately before, but almost at the same time as the red card.

**How Francisco (5:0) solves the classification: Inverse mode**

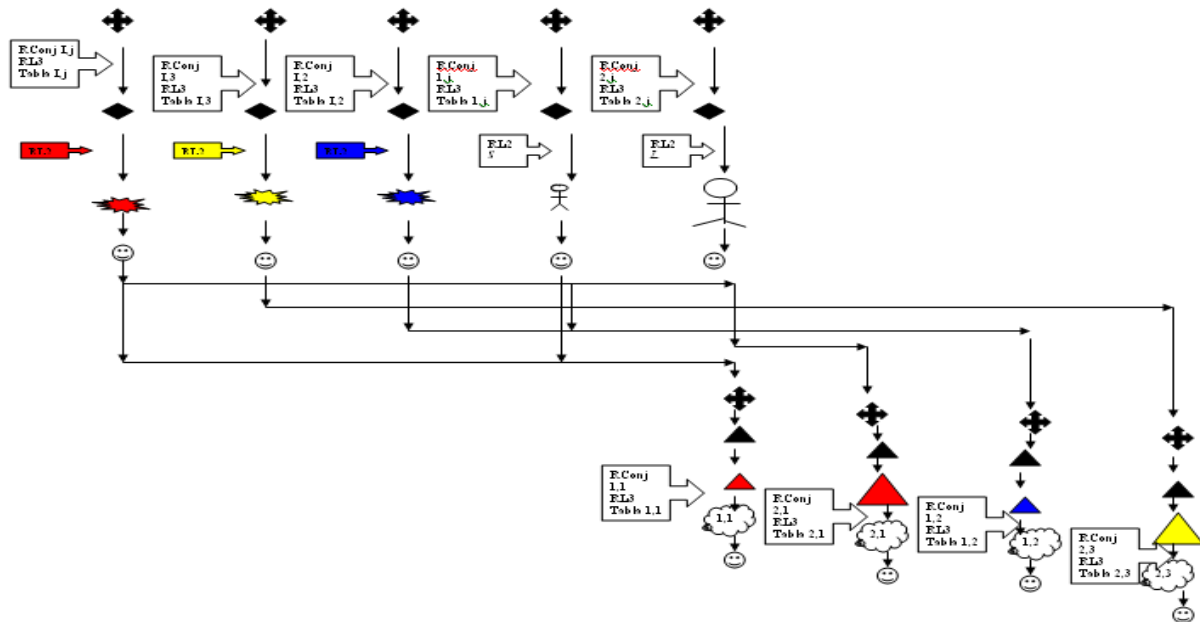


**Graph 6.-** How Francisco (5:0) solves the classification: Inverse mode

The child demonstrates a very regular procedure consisting in placing the cards indicating colour (beginning with those linked more to the pieces that he has as information), after the cards indicating size and finishes with the pieces. It might be thought that he places the red colour card unaided by other references and helped by the visual appearance of the grid of squares (Piaget 1975).

It can be seen in this procedure how the inverse mode contains the direct mode. In effect, once all the cards are positioned, the task is already equivalent to the direct mode.

### How Enrique (3:11) solves the classification: Inverse mode



**Graph 7.-** How Enrique (3:11) solves the classification: Inverse mode

He starts by placing the red colour card, the element that more than any other stresses inferential reasoning.

This child, who at the time had not yet reached the age of 4, uses a highly regular procedure, even greater than in the latter case, since, as may be seen in the graph, the positioning of the most specific elements (the four triangles) is done completing the columns, that is to say, it follows the colour code. The thought processes that it reveals demonstrate a regularity that is characteristic of mathematical thought to the extent that this is only applicable to regular phenomena.

It is notable that all the children who began the task in this way (21, or 10% of the total) successfully completed it.

## CONCLUSIONS

Relational trees allow the logical relations to be analysed that children establish on a step by step basis in the resolutive process as well as identifying the causes behind their errors. The results of the experiment, according to which the inverse mode is significantly more complex than the corresponding direct mode are explained by the greater relational complexity that is in turn reflected in the diagrams.

The complete lack of experience of the children in tasks presented in inverse mode leads us to speculate that the habitual practice of these tasks would allow the children to tackle them with greater success.

In mathematics the coexistence of two different but interrelated processes of thought to be highlighted: direct and inverse. However, the tasks in inverse mode should be standard practice in the preschool classroom. Moreover, the great success at the task in direct mode, recommended in some textbooks for children between 7 and 9 years old, underlines that the programming of activities designed to build up mathematical thought process at this stage should be reviewed.

## REFERENCES

- ALSINA, C., BURGUÉS, C., FORTUNY, J.M., GIMÉNEZ, J. AND TORRA, M. *Enseñar matemáticas*. Barcelona: Graó, 1996.
- DIENES, Z.P. *Los primeros pasos en matemática. Tomo I Lógica y juegos lógicos*. Barcelona: Teide, 1987.
- FLAVEL, J. H. *La psicología evolutiva de J. Piaget*. Barcelona: Paidós, 1982.
- PIAGET, J. *L'équilibration des structures cognitives*. París. Presses Universitaires de France, 1975.
- PIAGET, J. *Investigaciones sobre la abstracción reflexionante 1*. Buenos Aires: Huemul, 1979.
- POLYA, G. *Cómo plantear y resolver problemas*. México: Trillas, 1984.
- RUESGA, P. *El inicio del razonamiento en la infancia*. Burgos: Publicaciones de la Universidad de Burgos, 2004.
- SOLOW, D. *Como entender y hacer demostraciones en matemáticas*. México: Limusa, 1992.

## ANNEX I

## Classification. Direct mode

Interview	Observations	Analysis	Argumentation
<p>Interviewer (I).- How old are you?  Child (C).- Three.  I.- Let's see if you like the games that I've invented. You tell me later, eh?  I.- Look, do you know what this is...  (Sits down).  [Shows the cards for colour and sizes].  (Sits down).</p>			
<p>I.- Yes? Well, let's see, these are two dolls, aren't they?  C.- Yes  I.- But this one is...  C.- Small.  I.- Small, and this other one...  C.- Big.  I.- This...  [Goes over the cards indicating colour with a finger].  C.- Red... Yellow... Blue.  I.- That's right.</p>		<p>Explicitly identifies the values of the attribute 'size'.</p> <p>Explicitly identifies the values of the attribute 'colour'.</p>	
<p>I.- So I am going to place the cards here... Like that...  [The matrix is shown in the following way: vertically, large sizes above, small sizes below, horizontally, blue, yellow</p>			

Interview	Observations	Analysis	Argumentation
and red] (Sits down).			
I.- Good well look, now you have to put these triangles in their places, these are small and these are big... [The child has now identified the size] Each one in a square but pay attention to what the cards say.			
Picks up the large blue triangle and places it in (1,1)	Uses the first box of the matrix	Correct	
I.- Why do you put it there? N.- Because it is blue.			Expresses a value, corresponding to the colour of the shape
I.- And... what else? C.- And big.	The interviewer helps the child to take account of the other criterion.		
[Places the large red triangle on (1,3), the large yellow one on (1,2), the small yellow one on (2,2), the small red one on (2,3) and the small blue one on (2,1)]	Uses the last square and the square in the middle of the first row and then fills the second row. It seems that no order whatsoever is followed for the colour. Immediately, without any doubts	Correct It seems that the criteria 'size' is used to order the cards.	
I.- Why do you put this one here? [I points to the large yellow triangle on (1,2)] N.- Because it is big and yellow.			Expresses the value of the shape and of the colour



Interview	Observations	Analysis	Argumentation
<p>I.- And... this other one. Why do you put it there?            [I points to the small blue one (2,1)]            C.- Because it is small and blue.</p>			Expresses the value of the shape and of the colour

### Inverse mode

Interview	Observations	Analysis	Argumentation
<p>I.- We are going to play a guessing game. Look, I give you the cards because now you are going to have to put them in place.            [The child knows how to recognize the values shown on the cards]            I.- I am going to put the triangles in place. Only two, eh?. And you, paying a lot of attention to where I put them, you have to guess where you have to put the cards and the other triangles. O.K.?            (Looks with great attention and sits down).</p> <p>the small blue triangle is placed on (2,2) and the large yellow one on (1,3)].</p> <p>[C puts the red colour card in place].</p> <p>I.- And that one, why do you put it</p>		Starts placing the red colour card in a column without information.	

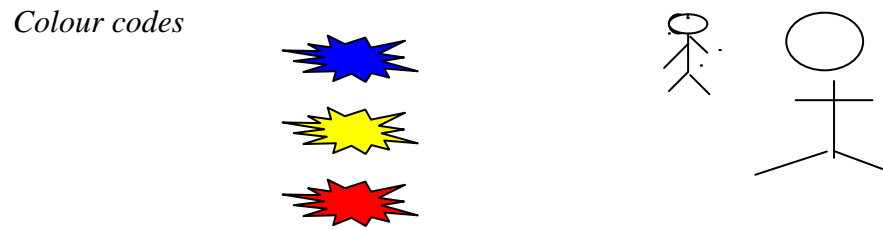
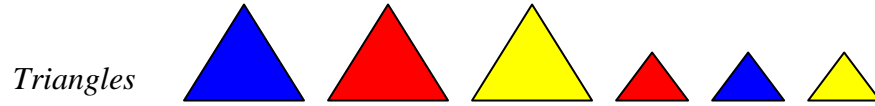
Interview	Observations	Analysis	Argumentation
there? (No reply)			
(He places the blue colour card on the yellow triangle and the yellow colour card on the blue triangle). (He puts the card for large sizes in place and underneath it the card indicating small sizes).		Uses the criteria of colour.  Manages to place the cards corresponding to the criterion size in order.	
I.- And that one, why do you put it there? (I points to the card for large sizes). C.- Because it's big.			Expresses a value of the criterion 'size'.
I.- Is that it? you'll leave it at that?. (Looks with great attention and sits down). (He remains in silence looking at the grid) I.- This one. Why is it here? [I points with a finger to the blue colour card]. C.- Because it's blue.	I prompts the child to inference on the actions.	Identifies the value on the card	
I.- Is it O.K. there? Shakes his head to say no and rapidly swaps the position of the yellow and the blue colour cards.	I prompts inference on the colour.	Manages to follow the prompt.	
[The child, up until now has only placed the cards] I.- So then, how do we put these triangles in place.			

Interview	Observations	Analysis	Argumentation
<p>[I points to the triangles that have not yet been used]  [C looks at the cards for each piece].  [Places the large red triangle on (1,1), the small red one on (2,1), the large blue one on (1,2) and the small yellow one on (2,3)]</p>	<p>Completes the grid following the columns.</p>	<p>Correct positioning.  It seems that he is following the criteria colour</p>	
<p>I.- And... this? Why do you put it here? (I points to the small yellow triangle on (2,3))  C.- Because the yellow one is here and the small one goes here.  [C points with a finger to the cards for yellow and small].</p>		<p>Identifies the two value of the piece</p>	<p>Expresses two values.</p>

**ANNEX II SYMBOLIZATION OF RELATIONAL DIAGRAMS**

**OBJECTS**

**Observables (Obs. O)**



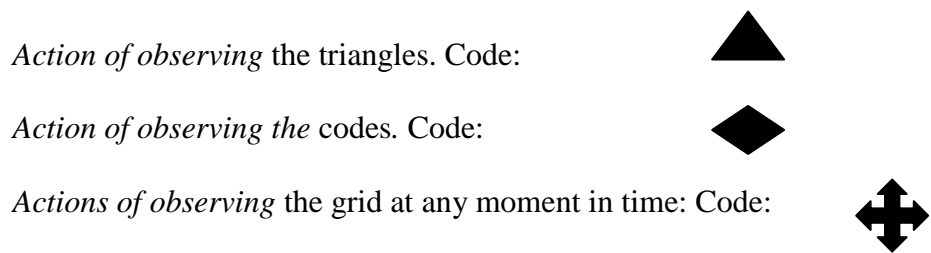
**Coordinates (Coord. O)**

**Grid of Squares**



**SUBJECT**

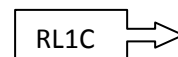
**Observables (Obs. S)**



**Coordinates (Coord. S)**

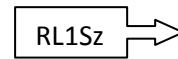
*Relations of appurtenance:* a triangle belonging to the class: -  
 Formed of triangles of the same colour.

Actions recognising this relation. Code:




Formed of triangles of the same size.

Actions recognising this relation. Code:




**Relations of appurtenance Level 2: a code belonging to a place on the grid:**

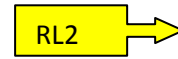
- Column of blue triangles (  )


Actions recognising this relation. Code:



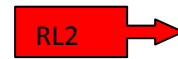
- Column of yellow triangles (  ).


Actions recognising this relation. Code:



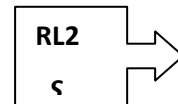
- Column of red triangles (  )

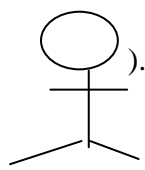
Actions recognising this relation. Code:



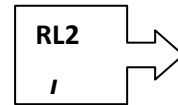
- Row of small triangles (  )

Actions recognising this relation. Code:



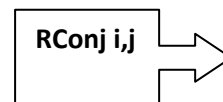
- Row of large triangles (  ).

Actions recognising this relation. Code:



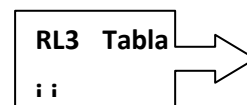
**Conjunctive relations:** Conjunction of values, of both attributes, shown by the grid for the square at row  $i$  and column  $j$ .

Actions recognising this relation. Code:



**Relations of appurtenance Level 3: a triangle belonging to a square at row  $i$ , column  $j$  of the grid:**

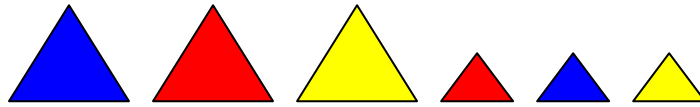
Actions recognising this relation. Code:



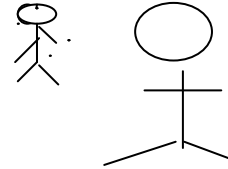
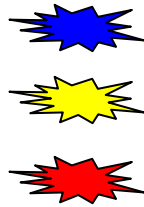
The chain of actions taken by the child are symbolized by the Code:



Actions selecting a particular triangle. Codes:



*Actions selecting each code of values. Codes:*



*Actions selecting.*

- row  $i$ , column  $j$  of the grid. Code:

- a place for a card. Code:



*Actions positioning an element in a square on the grid . Code:*



The end of a chain of actions (shown by the code:



at the end of the vertical chain ), the actions that follow are set out on the right.

Errors. Code:



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